

GENERAL THEORY
and
OPERATING INSTRUCTIONS
for
THE ELLIPSOMETER
by
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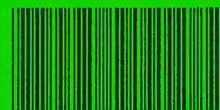
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The ellipsometer is an optical instrument which permits the determination of the optical thickness of thin films deposited onto reflecting surfaces. A film of an average thickness of $1 \text{ \AA}$, or so, can still be detected with the instrument.

When first confronted with this problem, one still shares the surprise of Lord Rayleigh who, more than sixty years ago, remarked that he "too hastily concluded that films of such extraordinary tenuity were unlikely to be of optical importance".

Suppose that plane polarized light is reflected from a surface and that the direction of vibration of the electric vector is at 45° to the plane of incidence. This electric vector OA can be looked upon as made up of two equal components, one OP, in the plane of incidence, and the other OS perpendicular to it. There are of course, two directions of vibration at 45° to the plane of incidence. This is represented in fig. 1 where the plane of the paper is perpendicular to the plane of incidence as well as to the direction of the light beam. If the reflecting body is non-metallic, after reflection both components OS and OP have suffered a 180° phase shift; this means that the reflected light is still plane polarized and vibrating in the same quadrants as before reflection. The orientation of the vibration, however, is not at 45° to the plane of incidence, since the component OS has become larger than OP, in other words, the angle between the plane of incidence and the vibration is more than 45° .

If the reflecting body is metallic, then the phase difference, Δ , which takes place at reflection between the two components OP and OS instead of being 0 is larger than 90° . The component OS is also larger than OP. Since there is a phase difference between the two components OP and OS, the reflected light is elliptically polarized. If the phase difference were exactly 90° , the ellipse characterizing the light would be oriented with its major axis perpendicular and its minor axis parallel to the plane of incidence. The component OP is in advance of OS with the result that the ellipse is dextro rotatory, if the vibration before reflection takes place in the second and fourth quadrant, whereas the ellipse is levo rotatory if the incident light vibrates in the first and third quadrants. See fig. 2. The different orientations of the ellipse, depending on the value of the phase difference Δ , have also been indicated in the figure.

Now, as first shown years ago by Drude, the presence of a thin transparent film on the reflecting surface will increase the ratio $\frac{OP}{OS}$ as well as decrease their phase difference $\Delta = \delta_{OP} - \delta_{OS}$. There is a quantitative relationship between these changes and the optical thickness of the film, which depends on the optical constants of the metal and the angle of incidence. In order to determine the changes in the ratio $\frac{OP}{OS}$ and in Δ , it is sufficient to determine the changes in the orientation as well as in the ratio of minor to major axis of the ellipse (ellipticity). Once the ellipse has been determined, simple algebraic relationships permit the calculation of the ratio $\frac{OP}{OS}$ as well as Δ . The ellipse can be determined by compensating the phase retardation of 90° which occurs between the components along the major and minor axes. This is accomplished by placing a $\lambda/4$ plate in the path of the reflected light, with the principal directions along the axes of the ellipse. Thus, when the $\lambda/4$ plate is properly oriented, the light comes out plane polarized and can be extinguished with a crossed analyzing nicol.

The correctly oriented $\lambda/4$ plate locates the axes of the ellipse. The tangent of the angle made by the direction of the reestablished plane polarized light with the major axis of the ellipse gives the ellipticity (ratio of minor to major axis). Qualitatively, the presence of a film will change the orientation of the ellipse as well as the ellipticity. The axes rotate counterclockwise if the incident light is in the second and fourth quadrants and clockwise if the light vibrates in the first and third quadrants. In both cases, the ellipse becomes "fatter" (increase in the ellipticity).

The essential parts of the ellipsometer are:

- 1) The polarizer, which can rotate along an axis parallel to the light beam and can be placed with the direction of the transmitted vibration in either one of four fixed positions, one in the plane of incidence, one perpendicular to the plane of incidence, and the other two at 45° to the plane of incidence.
- 2) A rotating stage supporting the slide, upon which the films are deposited. The rotation of the stage varies the angle of incidence.

3) A $\lambda/4$ plate followed by an analyzer. Both the $\lambda/4$ plate and analyzer can rotate along an axis parallel to the light beam and their respective orientations can be read on one or two divided circles (depending on the model of the instrument). On the one circle model, the $\lambda/4$ plate is attached to an arm with a movable vernier. Its position is read on the circle which is kept fixed in a reference position.

4) A photometer equipped with a photomultiplier tube to measure the amount of light emerging from the analyzer.

For the determination of the optical thickness of a film, the position of both $\lambda/4$ plate and analyzer corresponding to a minimum in the transmitted light are noted, when the light is reflected first from a clean slide and again after the film has been deposited.

The rotation of the $\lambda/4$ plate (levo or dextro depending on whether the light transmitted by the polarizer was in the second and fourth or first and third quadrants) measures the rotation of the ellipse. The angle turned by the analyzer can be looked upon as the sum of two angles. Firstly, the angle of rotation of the ellipse. Secondly, the difference in the angles whose tangent is the ellipticity after and before the presence of a film. If the film did not affect the ellipticity but would produce a change in Δ only, the rotation of the analyzer would be equal to that of the $\lambda/4$ plate.

It should be made quite clear that the sensitivity of the ellipsometer is given by the magnitude of the angle turned by the direction of re-established plane polarized light (as measured by the rotation of the analyzer) after a film has been deposited on the reflecting surface. As it has just been said this angle can be looked upon as the sum of two angles. These angles can be either in the same direction, or in opposite direction depending on the orientation of the $\lambda/4$ plate in the compensating position as it will be shown presently. Suppose the electric vector of the incident light vibrates in the second and fourth quadrant and that the phase difference of 90° between the two axes of the ellipse obtained after reflection is brought to zero by making the fast direction of the plate coincide with the direction OA of the major axis of the ellipse, which is retarded by 90° with respect to OB. In this case after the deposition of the film the two angles $(\alpha' - \alpha)$ and $(\psi' - \psi)$ are in the same direction. See figures O_I and O_{II} . The analyzer has to be turned by $(\alpha' - \alpha) + (\psi' - \psi)$ to get back extinction after the deposition

of the film. The major axis of both ellipses, representing the reflected light before and after deposition of a thin film are in the first and third quadrance since for metallic reflection at $\sim 70^\circ$ incidence the phase difference OP-OS is between 90° and 180° . (This would not be the case of ellipse O_{II} if the film were relatively thick).

Let us now orient the $\lambda/4$ plate with its fast direction along the minor axis OB of the ellipse, which is in advance by 90° with respect to OA. The phase difference OB-OA is now 180° instead of 0° . As it is apparent from the figures O_{III} and O_{IV} , after the deposition of the film, the angle $(\psi' - \psi)$ and $(\alpha' - \alpha)$ are in opposite direction. The analyzer has to be turned by $(\alpha' - \alpha) - (\psi' - \psi)$ instead of $(\alpha' - \alpha) + (\psi' - \psi)$ in order to re-establish extinction after compensation of the ellipse. For instance, a film of Ba stearate $48 \text{ \AA}$ thick, at 69° incidence brings a rotation of the ellipse of $(\alpha' - \alpha) = 2.3$ and a corresponding change $(\psi' - \psi) = 3^\circ$. Consequently, when the $\lambda/4$ is oriented with its fast direction along OA, the analyzer has to be turned by $2.3 + 3 = 5.3$, to re-establish extinction after the deposition of the film. When the $\lambda/4$ is oriented with its fast direction along OB, the analyzer has to be turned by $2.3 - 3.0 = 0.7^\circ$ to re-establish extinction. Thus, in the second case the sensitivity has been reduced considerably. Therefore, the $\lambda/4$ plate should be oriented with its fast axis along OA to obtain the maximum sensitivity.

The simplest way to correlate optical thickness of films with the angles of rotation of $\lambda/4$ plate and analyzer is to prepare empirical calibration curves with films of known thickness. Films of Ba stearate which can easily be transferred onto slides by the method of Blodgett and Langmuir are very useful for such calibration. For films from 0 to $700 \text{ \AA}$ thick, the plot of thickness versus analyzer position is sufficient for optical thickness determination of unknown films. However, the position of the analyzer and $\lambda/4$ plate necessary to obtain a minimum in the intensity of the transmitted light, is a periodic function (not sinusoidal) of the film thickness. The analyzer reaches a series of maxima and minima positions as the film thickness increases. The first maximum (or minimum) occurs around $800 \text{ \AA}$, and the second one around $1170 \text{ \AA}$. Film thickness determination would be ambiguous in these critical ranges, if the position of the analyzer was the only information available. However, since the minimum or maximum position of the $\lambda/4$ plate does not occur in the same thickness range as the corresponding minimum or

maximum of the analyzer, a second calibration curve plotting thickness against position of the $\lambda/4$ plate will permit an unambiguous thickness determination.

For very thin films deposited onto chromium plated slides, the rotation of the analyzer is 0.083° per angstrom of film at 69° incidence. The sensitivity obtained with carbide gauge blocks is about 30% less than that with chromium slides, thus showing the great importance of the optical constants of the metallic substrate.

If the refractive index of the film is different from that of Barium stearate, adequate correction can be applied to convert the optical thickness to the actual thickness with the use of the following Drude equation, valid only for very thin films:

$$\delta\Delta = -A(1 - \frac{1}{n^2})\ell$$

Where A is a constant, $\delta\Delta$ is the phase shift produced by the film of refractive index n and thickness l.

Determination of the direction of the electric vector transmitted by the polarizer.

As described elsewhere, the polarizer can be set in four fixed positions at 45° to each other. One is in the plane of incidence, one in the plane perpendicular to it and the other two are at 45° . If there is any doubt about the direction of vibration transmitted by the polarizer, this can easily be checked in the following way. Take a small piece of polaroid plate and note the direction in which one has to orient it in order to obtain a minimum in the light intensity of a source reflected from a glass plate at the Brewsterian angle and viewed through the Polaroid plate. Near the Brewsterian angle ($\simeq 56^\circ$) the reflected light is plane polarized with the electric vector vibrating perpendicularly to the plane of incidence. The reflected light viewed through the Polaroid plate will reach a minimum when the direction transmitted by the Polaroid will be in the plane of incidence. The direction of vibration transmitted by the polaroid being thus established, the direction of vibration transmitted by the polarizer in a given position is easily determined by crossing polarizer with polaroid.

The $\lambda/4$ plate.

It rarely occurs that the retardation produced by the so called $\lambda/4$ plate is exactly 90° . Furthermore, a plate can be exactly $\lambda/4$

for one wave length only. For the practical determination of the optical thickness of surface films it is of no importance whether the plate is exactly $\lambda/4$ or not, except if the ellipticity of the reflected light is close to 1, as we shall see later. (By definition, an ellipticity of 1 corresponds to circularly polarized light.) All that is required is that the light emerges plane polarized after passage through the $\lambda/4$ plate, and this can be accomplished even with a plate whose retardation is not exactly 90° , as the following considerations will show. For any ellipse characterizing the reflected light, there is a 90° phase difference only along the components of the major and minor axis of the ellipse. For any other pair of components, mutually perpendicular, their phase difference Δ , is either less or more than 90° . In the case of a dextro ellipse (an ellipse described in a clockwise fashion), the phase difference Δ between the components a and b is larger than 90° if the components a and b are located to the left of the minor and major axis respectively. The phase difference is smaller than 90° if the components a and b are to the right of the minor and major axis. The situation is reversed in the case of a levo ellipse. These facts are illustrated in Fig. 3 in the case of a dextro ellipse. The phase difference of any pair of mutually perpendicular components Ob and Oa, the ellipticity ξ (the angle whose tangent is equal to the ratio of the minor to major axis of the ellipse $\frac{OB}{OA}$), and θ , the angle whose tangent is the ratio of the components Ob and Oa, are all correlated by the following equation:

$$(1) \quad \sin \Delta = \sin 2\xi / \sin 2\theta$$

Equation (2) correlates the phase difference Δ of any pair of mutually perpendicular components Ob and Oa, the ellipticity ξ , and β , the angle made by the components Oa or Ob with the major or minor axis of the ellipse:

$$(2) \quad \tan \Delta = \tan 2\xi / \sin 2\beta$$

Generally, the angle β refers to the angle made by the major axis of the ellipse and the component Oa of any pair of mutually perpendicular components, Oa and Ob, as indicated in Fig. 3. However, when these components are those along the principal directions of the $\lambda/4$ plate, we will call the angle β_p (as in Fig. 6); and when these components are those along the plane of incidence and its perpendicular, we shall call the angle β_i (as in Fig. 7). From equations (1) and (2), it is apparent that Δ reaches a minimum when $\sin 2\theta = \sin 2\beta = 1$, that is, when the components Oa and Ob are equal and thus inclined at 45°

to the axis of the ellipse. See diagram III of Fig. 3. It is now clear that in order to compensate a given ellipse, the smaller the true value of the $\lambda/4$ plate, the larger the angle β must be. In other words, the smaller the value of the $\lambda/4$ plate, the larger is the angle between one of its principal directions and the axis of the ellipse. However, if the ellipticity were large and the $\lambda/4$ plate were far from 90° it might happen that the ellipse could not be compensated. The phase retardation of the plate must be at least twice the ellipticity if the ellipse is to be compensated. This limiting condition is given by equation (2) in which $\beta = 45^\circ$. The same conditions hold when the plate is greater than 90° ; for instance, in the case of a dextro ellipse (see diagram I Fig. 3), the angle β is to the left of the major axis OA, consequently negative, hence $\tan \Delta$ is negative (see equation (2)) and the angle Δ is the supplement of the angle Δ found had the angle β been positive. In general, for very thin films deposited onto metallic surfaces, the ellipticity is considerably less than unity and no difficulty is encountered in compensating the ellipse even if the plate is appreciably smaller or larger than 90° . However, in the range of thickness around 600A the ellipticity of the reflected light is close to 1; the light is nearly circularly polarized and the plate retardation has to be as close as possible to 90° in order to achieve complete compensation.

Determination of the phase retardation produced by the $\lambda/4$ plate.

It is first necessary to determine the position of both analyzer and $\lambda/4$ plate for which the light transmitted by the polarizer in a given position is completely extinguished. The observation is made with the light source, polarizer, $\lambda/4$ plate, analyzer and telescope in alignment (no reflection). When the extinction has been accomplished, it means that the principal directions of the $\lambda/4$ are parallel and perpendicular respectively to the direction transmitted by the polarizer and that the analyzer is crossed with the polarizer. These positions of the $\lambda/4$ plate and analyzer are recorded as reference positions. It is still unknown which of the two principal directions of the $\lambda/4$ plate is the fast or the slow one. However, this can easily be determined as will be shown in the next paragraph.

If the ellipsometer has only one divided circle, the position of the analyzer is read directly on the divided circle with respect to a fixed vernier. In the one circle model, the $\lambda/4$ plate is connected with a movable vernier, and the position of the plate is also referred

to the scale of the circle; but it should be remembered that the reading on the movable vernier should be taken with the divided circle always in the same position with respect to the fixed vernier.

The following steps are now taken for the calculation of the phase retardation produced by the plate.

- 1) Place the polarizer with the direction of transmission at 45° to the plane of incidence. Let us assume that the vibration takes place in the second and fourth quadrants.
- 2) Place a metallic reflecting slide - a chromium plated glass slide for instance - on the stage and set the angle of incidence to about 70° . At this incidence, it is known that the phase difference which takes place on reflection between the components parallel and perpendicular to the plane of incidence is more than 90° , the component parallel to the plane of incidence being in advance and smaller than the component perpendicular to it. With the incident light vibrating in the second and fourth quadrants, the ellipse characterizing the reflected light is dextrorotatory, its major axis is oriented in the first and third quadrants and is closer to the perpendicular to the plane of incidence than to the plane of incidence.

The $\lambda/4$ plate and analyzer are rotated until their positions are such that a minimum is reached in the intensity of the reflected light. When this is achieved, it means that the light emerging from the plate has become plane polarized (the ellipse is said to have been compensated), and has been extinguished by the crossed analyzer.

Now, there are two possible compensating orientations for the $\lambda/4$ plate. If its phase retardation is exactly $\lambda/4$, the two positions are exactly 90° apart; in one of them the fast direction of the plate coincides with the major axis of the ellipse (first and third quadrants) and since the ellipse is dextrorotatory, the phase difference is brought back to zero. For the particular ellipticity and orientation of the ellipse drawn in Fig. 4a, the resulting linear vibration is located in the second and fourth quadrants. In the other compensating orientation, the slow direction of the plate coincides with the major axis of the dextro ellipse and the phase difference becomes 180° . The resulting linear vibration is located in the first and third quadrants (see Fig. 4b).

Thus, the location of the reestablished linear vibration corresponding to the two compensating positions of the plate will tell which one of the two principal directions of the plate coincides with the major (or minor) axis of the ellipse. This information permits the identification of the



fast and slow directions for the records of the reference position of the $\lambda/4$ plate as mentioned at the beginning of this section on the determination of the phase retardation produced by the $\lambda/4$ plate.

If the retardation produced by the plate is not exactly 90° , the principal directions of the plate do not coincide with the axes of the ellipse in the two compensating positions. If the ellipse is dextro-rotatory and the plate more than $\lambda/4$, the principal directions of the plate have turned slightly counterclockwise when the phase shift is brought to zero, and slightly clockwise when the phase shift is brought to 180° .

This is illustrated in Fig. 3, where the fast direction of the plate would correspond to the component Oa of diagram I and the slow direction of the plate would correspond to the component Oa of diagram II. If the plate is less than $\lambda/4$, the rotations of the plate to achieve compensation are opposite to those first described.

Thus, the angles made by the axis of the ellipse and the direction of the plate, the fast one in one compensating position and the slow one in the other, are equal. This means that the axis bisects the angle made by the principal direction in the two compensating positions; half the value of this angle made by one direction of the plate and the axis of the ellipse we shall call β_0 . Thus, even with a plate not exactly $\lambda/4$, the orientation of the ellipse can be accurately determined by taking the two positions of the plate which compensate the ellipse.

If the $\lambda/4$ plate were exactly 90° , the ellipticity would be given by the tangent of the angle made by the direction of vibration of the reestablished plane polarized light (indicated by the analyzer) and the fast or slow direction of the plate, depending on whether the phase difference was brought back to zero or to 180° , as illustrated in Fig. 5:

Since the plate is not exactly $\lambda/4$, and consequently not oriented along the axes of the ellipse in the compensating positions, the angle made by the direction of vibration of the reestablished plane polarized light with one of the directions of the plate is not the ellipticity ξ , but has the value θ , whose tangent is the ratio of the components of the ellipse along the principal directions of the plate as indicated in Fig. 6:

Knowing β and θ , the ellipticity can be calculated with equation (3):

$$(3) \quad \tan^2 \xi = \frac{\sin^2 \beta - \tan^2 \theta \cos^2 \beta}{\tan^2 \theta \sin^2 \beta - \cos^2 \beta}$$

where β in this formula has the value β_p .

In practice, if the $\lambda/4$ plate is not much different from 90° , the ellipticity can be taken as equal to the experimental value of θ .

The ellipticity ξ and the angle β being determined, the phase difference Δ occurring between the components of the ellipse along which are oriented the principal directions of the plate is given by equation (2) where β has the value of β_p .

This value of Δ is thus equal to the value of the $\lambda/4$ plate. It should be remembered that if the fast direction of the plate in the compensating position is turned counterclockwise with respect to the major axis of a dextro ellipse (original vibration in second and fourth quadrants), it means that the plate is more than $\lambda/4$ and its value is $180^\circ - \Delta$, Δ being the value given by equation (2).

Verification of the proper adjustment of the polarizer.

It can be taken for granted that the four fixed positions of the mounting holding the polarizing prism have been accurately machined 45° apart. One has to verify however, that the prism has been correctly mounted within its frame, and that the direction of the transmitted vibration coincides with one of the four positions. It has been shown in the preceding section how the value of the $\lambda/4$ plate can be calculated by determining the ellipticity and orientation of the ellipse corresponding to the light reflected from a metallic slide, when the incident light is plane polarized at 45° to the plane of incidence. It had been assumed that the incident light was vibrating in the second and fourth quadrants. Now the polarizer is placed in the other 45° position, with the electric vector vibrating in the first and third quadrants. The ellipticity and orientation of the levo ellipse thus obtained is determined as previously described for the dextro ellipse by carrying the compensation for the two positions of the $\lambda/4$ plate. If the polarizer is correctly adjusted, the ellipses characterizing the reflected light for both 45° positions have the same ellipticity, and are symmetrically oriented with respect to the plane of incidence or its perpendicular. The dextro ellipse has its major axis turned clockwise with respect to the perpendicular to the plane of incidence

and the levo ellipse has its major axis turned counterclockwise by the same angle which we shall call β_i . This is illustrated in Fig. 7, where the two compensating positions of the plate have been indicated, assuming that the compensating plate was slightly more than 90° .

With the incident vibration in the second and fourth quadrants, let us assume that the polarizing prism is not correctly adjusted and that the angle made by the vibration and the perpendicular to the plane of incidence is larger than 45° . Consequently, this angle will be smaller than 45° when the polarizer is turned to the other 45° position. It follows that before reflection in the first case the component parallel to the plane of incidence is larger than the perpendicular component, whereas the situation is reversed in the second case. Due to this lack of equality of the components the ellipse corresponding to the two approximately 45° positions of the polarizer obtained after reflection, will not have the same ellipticity, and will not be symmetrically located with respect to the plane of incidence or its perpendicular.

The phase difference between the components parallel and perpendicular to the plane of incidence is constant for a given slide at a given incidence, whatever the value of the ratio of the components; in other words, this phase difference is independent of the adjustment of the polarizer. If one takes Δ in equation (2) as this phase difference, and β as the angle between the major axis of the ellipse of ellipticity ξ and the perpendicular to plane of incidence (β_i), it follows that in order to keep Δ constant, β must change in the same directions as ξ .

$$\tan \Delta = \frac{\tan 2\xi}{\sin 2\beta_i} = \text{constant}$$

With our assumption that the direction of vibration of the incident light is in the second and fourth quadrants and makes an angle larger than 45° with the perpendicular to the plane of incidence, the ellipticity of the resulting dextro ellipse is larger than it would have been had the polarizer been correctly set at 45° . Consequently, the angle β_i is also larger. Since the major axis of this dextro ellipse is in the first and third quadrants, the axis will have turned clockwise. With the polarizer turned by 90° , the vibration is in the

first and third quadrants, and the angle made by the vibration and the perpendicular to the plane of incidence is smaller than 45° . The ellipticity of the resulting levo ellipse is smaller and the angle β_i is also smaller than the values obtained with a correctly set polarizer. Since the major axis of this levo ellipse is located in the second and fourth quadrants, it means that the major axis will also have turned clockwise. As a first approximation, it can be said that the angles $\Delta\beta_i$ turned by the axes of both ellipses from the positions they would have had with a correctly adjusted polarizer, are equal.

The axis of each ellipse bisects the angle made by the two principal directions in the two compensating positions of the plate, as we have already seen. Experimentally, the angles β_i made by the major axis of each ellipse and the uncorrected perpendicular to the plane of incidence are determined for both 45° positions of the polarizer. These two angles are not the same, and following our assumption, the angle obtained for the dextro ellipse (β_i^d) is larger than the angle (β_i^l) for the levo ellipse. A consideration of Fig. 8 will show that equation (4) should hold between β_i^d , β_i^l , $\Delta\beta_i$ and γ , where γ is the angle by which the polarizer should be rotated within its frame to be correctly adjusted:

$$(4) \quad \Delta\beta_i + \beta_i^l + \gamma = \beta_i^d - \Delta\beta_i - \gamma$$

$$\Delta\beta_i + \gamma = \frac{\beta_i^d - \beta_i^l}{2}$$

Furthermore, the ellipticity of both ellipses ξ^d and ξ^l have been determined as described above and using equation (2) one can write the following:

$$\frac{\tan 2\xi^d}{\tan 2\xi^l} = \frac{\sin 2\beta_{i_0}^d}{\sin 2\beta_{i_0}^l}$$

where β_{i_0} refers to the angles made by the axes of the ellipses with the true perpendicular to the plane of incidence.

$$\text{since } \beta_{i_0}^d = \beta_i^d - \gamma \quad \text{and} \quad \beta_{i_0}^l = \beta_i^l + \gamma$$

$$(5) \text{ hence } \frac{\tan 2\xi^d}{\tan 2\xi^l} = \frac{\sin 2(\beta_i^d - \gamma)}{\sin 2(\beta_i^l + \gamma)}$$

The simplest way to obtain γ is to estimate a value which satisfies the above equation. Once γ has thus been determined, $\Delta\beta_i$ is obtained from:

$$\Delta\beta_i = \frac{\beta_i^d - \beta_i^l}{2} - \gamma$$

Calculation of the value of the phase difference Δ occurring between the true directions parallel and perpendicular to the plane of incidence.

This value is easily obtained since the β_{i_0}' 's values are known.

$$\tan \Delta = \frac{\tan 2\xi^d}{\sin 2\beta_{i_0}^d} = \frac{\tan 2\xi^e}{\sin 2\beta_{i_0}^e}$$

Calculation of the parameters θ of both ellipses. θ is the ratio of the components parallel and perpendicular to the plane of incidence.

Using equation (1) one gets:

$$\sin 2\theta^d = \frac{\sin 2\xi^d}{\sin \Delta}, \quad \sin 2\theta^e = \frac{\sin 2\xi^e}{\sin \Delta}$$

knowing θ^d and θ^e , one can calculate the ratio:

$$\frac{\tan \theta^d}{\tan \theta^e}$$

which can also be obtained in the following way:

Before reflection, when the light vibrates in the second and fourth quadrants, according to our assumption, the component parallel to the plane of incidence is $A \cos (45^\circ - \gamma)$ or $A \sin (45^\circ + \gamma)$, and the component perpendicular to it is $A \cos (45^\circ + \gamma)$, where A is the magnitude of the incident electric vector.

$$\text{There ratio is } \theta_1 = \frac{\sin (45^\circ + \gamma)}{\cos (45^\circ + \gamma)} = \tan (45^\circ + \gamma)$$

When the polarizer is turned 90° with the light vibrating in the first and third quadrants, for the same ratio of the components parallel and perpendicular to the plane of incidence, one gets:

$$\theta_2 = \frac{\sin (45^\circ - \gamma)}{\cos (45^\circ - \gamma)} = \tan (45^\circ - \gamma)$$

After reflection, if one calls θ the ratio which would have been obtained had the polarizer been correctly adjusted one gets:

$$\tan \theta^d = \tan \theta \tan (45^\circ + \gamma) \text{ and } \tan \theta^e = \tan \theta \tan (45^\circ - \gamma)$$

their ratio:

$$\frac{\tan \theta^d}{\tan \theta^e} = \frac{\tan (45^\circ + \gamma)}{\tan (45^\circ - \gamma)}$$

must be equal to the one calculated at the beginning of the section.

Numerical Application for the determination of the value of the $\lambda/4$ plate and the adjustment of the polarizer.

The theoretical considerations presented above will now be illustrated by an actual numerical case.

The light is reflected from a chromium plated slide at approximately 70° incidence, with the polarizer set with the incident vibration at 45° in the second and fourth quadrants. It is found by following the steps described in the preceding procedure that in one compensating position the fast direction of the plate makes an angle of $18^\circ 30'$ to the right of the approximate perpendicular to the plane of incidence, and that the analyzer makes an angle of $1^\circ 42'$ to the left of the same perpendicular. (As said previously these positions of $\lambda/4$ plate and analyzer are obtained by turning plate and analyzer until a minimum is reached in the reflected light.) In the other compensating position, the slow direction makes an angle of $18^\circ 48'$ to the right of the approximate perpendicular, and the analyzer makes an angle of $38^\circ 57'$ to the right of the same perpendicular. From the values of the two angles made by the principal direction (fast in the first case and slow in the second) with the approximate perpendicular to the plane of incidence, it is apparent that the major axis of the dextro ellipse characterizing the reflected light is located $18^\circ 39'$ to the right of the approximate perpendicular to the plane of incidence. Since the ellipse is dextrorotatory, and the fast direction of the plate slightly to the left of the major axis of the ellipse in one of the compensating positions, it can be concluded that the plate produces a phase shift slightly greater than 90° . The angle β_p made by the axis of the ellipse and the principal direction of the plate being only $9'$ ($18^\circ 39' - 18^\circ 30'$), it can be assumed that the ellipticity of the ellipse is given by the ratio of the two components vibrating along the principal directions of the plate. The angle whose tangent is the ellipticity is thus $18^\circ 30' + 1^\circ 42' = 20^\circ 12'$ from the values obtained in the first compensating position, and $38^\circ 57' - 18^\circ 48' = 20^\circ 9'$ from the values of the second compensating position. These two values for the ellipticity $\mathcal{E}$ are in satisfactory agreement.

From the values of $\beta_p = 9'$ and $\mathcal{E} = 20^\circ 10'$ one gets using equation (2)

$$\tan \Delta = \frac{\tan 2\mathcal{E}}{\sin 2\beta} = \frac{\tan 40^\circ 20'}{\sin 18'} ; \quad = 89^\circ 39'$$

But, since the plane retardation of the plate, as we have seen, is larger than 90° , Δ is actually the supplement of $89^\circ 39'$; that is, $90^\circ 21'$. This is the value of the $\lambda/4$ plate.

When the light vibrates in the other 45° position, i.e., in the first and third quadrants, it is found that, in one compensating position, the fast direction makes an angle of $16^\circ 47'$ to the left of the apparent

perpendicular to the plane of incidence and the analyzer makes an angle of $2^\circ 56'$ to the right of the same direction. It can be assumed as a first approximation that the ellipticity is $19^\circ 43'$. In the other compensating position, the slow direction makes an angle of $17^\circ 6'$ to the left of the apparent perpendicular to the plane of incidence and the analyzer makes an angle of $36^\circ 32'$. The ellipse is levorotatory, and since the $\lambda/4$ plate is more than 90° (as we have already seen), the fast direction should be to the right of the major axis of the ellipse, in one compensating position, and the slow direction should be to the left of the major axis, in the other compensating position. This is borne out by the figures just given. The major axis of this levo ellipse thus makes an angle $16^\circ 55'$ to the left of the apparent perpendicular to the plane of incidence.

As we have said previously, when a light beam is polarized at exactly 45° to the plane of incidence, the component perpendicular to the plane of incidence is greater than the component parallel to the plane of incidence after reflection. The ellipticity of the levo ellipse being $19^\circ 42'$ and that of the dextro ellipse $20^\circ 10'$, it follows that before reflection when the light vibrates in the second and fourth quadrants the component vibrating in the plane of incidence is greater than the same component when the light vibrates in the first and third quadrants. In other words, the angle made by the electric vector with the perpendicular to the plane of incidence is greater than 45° when the vibration takes place in the second and fourth quadrants and less than 45° when the vibration takes place in the first and third quadrants.

Putting in equation 4 (see also fig. 8):

$$\Delta\beta_i + \beta_i^e + \gamma = \beta_i^d - \Delta\beta_i - \gamma$$

$$\beta_i^d = 18^\circ 39' \text{ and } \beta_i^e = 16^\circ 55'$$

$$\text{one gets } \Delta\beta_i + \gamma = \frac{1^\circ 44'}{2} = 52'$$

where γ is the angle by which the polarizer is off from the 45° position, and $\Delta\beta_i$ is the angle turned by the axis of each ellipse from the positions they would have had with a correctly adjusted polarizer.

From equation (5) one gets:

$$\frac{\tan 2\xi^d}{\tan 2\xi^e} = \frac{\sin 2(\beta_i^d - \gamma)}{\sin 2(\beta_i^e + \gamma)}$$

$$\frac{\tan 2 \times 20^\circ 10'}{\tan 2 \times 19^\circ 42'} = \frac{\tan 40^\circ 20'}{\tan 39^\circ 24'} = \frac{\sin 2(18^\circ 39' - \gamma)}{\sin 2(16^\circ 55' + \gamma)}$$

The above equation is satisfied by choosing $\gamma = 31'$. It follows that

$$\Delta\beta_i = 21'.$$

The phase difference between the two components parallel to the true plane of incidence and its perpendicular is thus

$$\tan \Delta = \frac{\tan 2 \times 20^{\circ} 10'}{\sin 2 (18^{\circ} 39' - 31')} = \frac{\tan 40^{\circ} 20'}{\sin 36^{\circ} 16'}$$

$$\Delta = 55^{\circ} 8'$$

But, since the value of Δ is more than 90° , the true angle is the supplement of $55^{\circ} 8'$ that is $124^{\circ} 52'$. Obviously, the same value is obtained within experimental error by making use of the figures corresponding to the levo ellipse.

The ratio θ^d of the components parallel and perpendicular to the plane of incidence of the dextro ellipse is given by:

$$\sin 2\theta^d = \frac{\sin(2 \times 20^{\circ} 10')}{\sin 124^{\circ} 52'} = \frac{\sin 40^{\circ} 20'}{\sin 55^{\circ} 8'}$$

$$\theta^d = 26^{\circ} 2'$$

The ratio of the same components for the levo ellipse is given by:

$$\sin 2\theta^e = \frac{\sin(2 \times 19^{\circ} 42')}{\sin 124^{\circ} 52'} = \frac{\sin 39^{\circ} 24'}{\sin 55^{\circ} 8'}$$

$$\theta^e = 25^{\circ} 19'$$

The ratio: $\frac{\tan \theta^d}{\tan \theta^e} = 1.033$ is equal within experimental

error to the ratio:

$$\frac{\tan 45^{\circ} + 31'}{\tan 45^{\circ} - 31'} = 1.035$$

as it should be according to the theoretical consideration of the previous section.

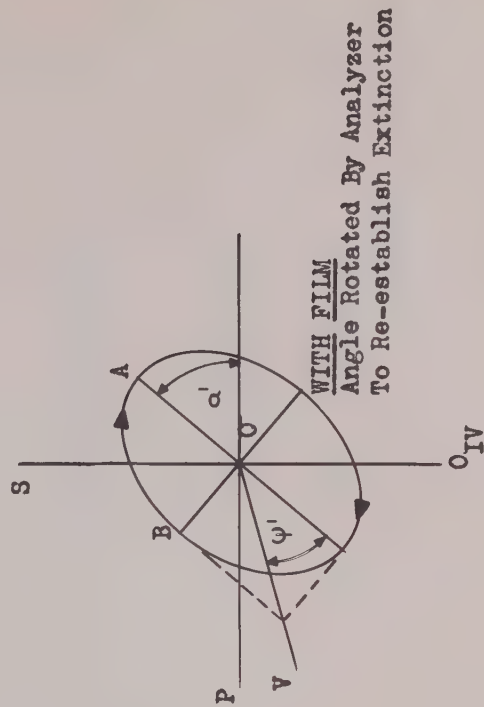
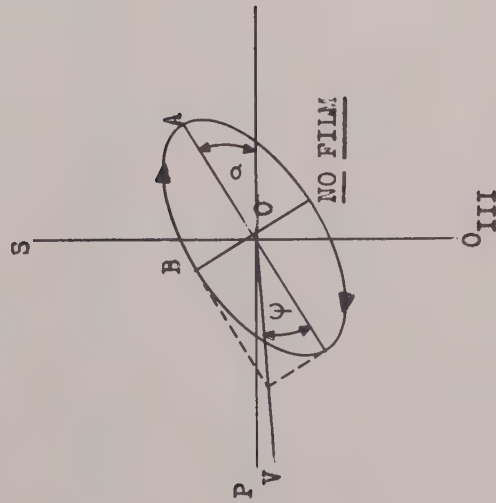
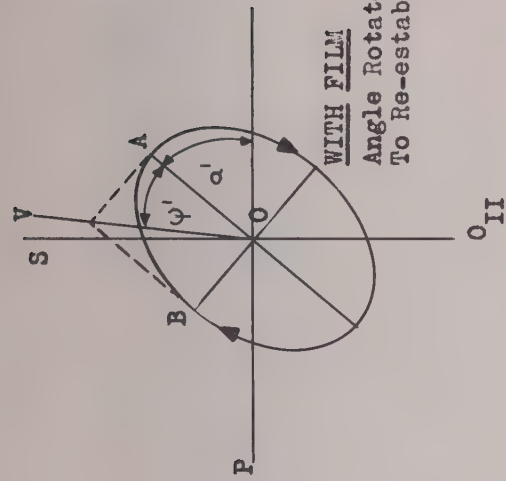
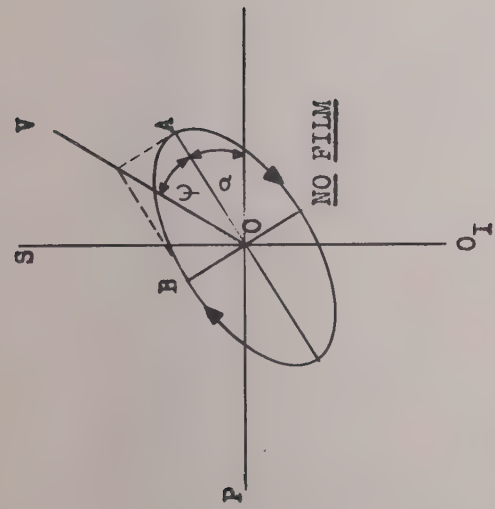


FIGURE-0

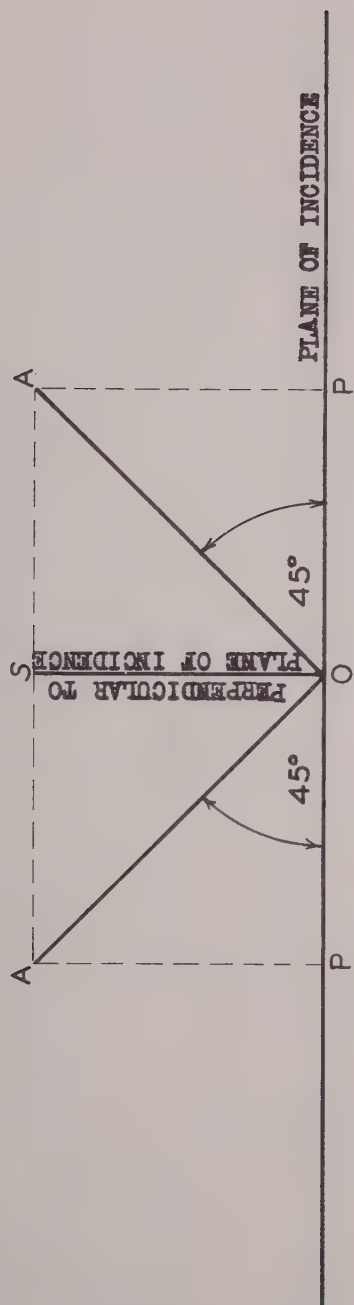
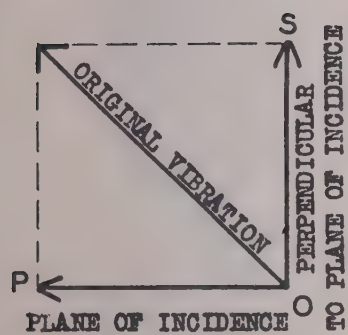


FIGURE 1

INCIDENT VIBRATION
IN THE 2nd AND 4th
QUADRANTS



RESULTING
REFLECTED
VIBRATION

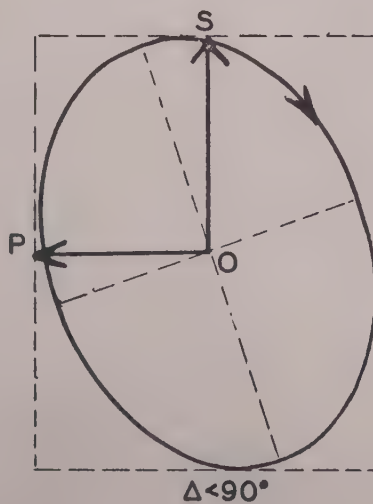
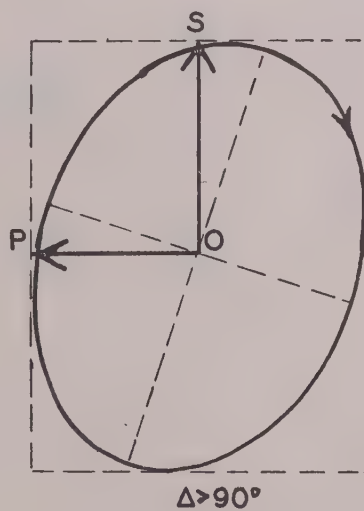
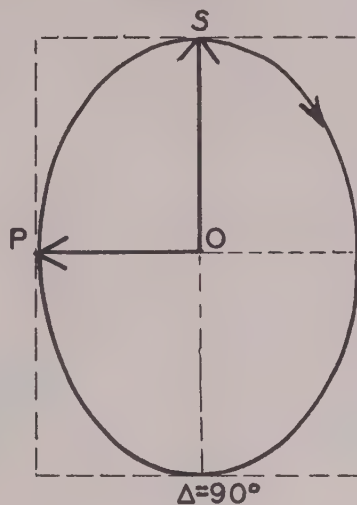


FIGURE 2

INCIDENT VIBRATION
IN THE 1st AND 3rd
QUADRANTS



RESULTING
REFLECTED
VIBRATION

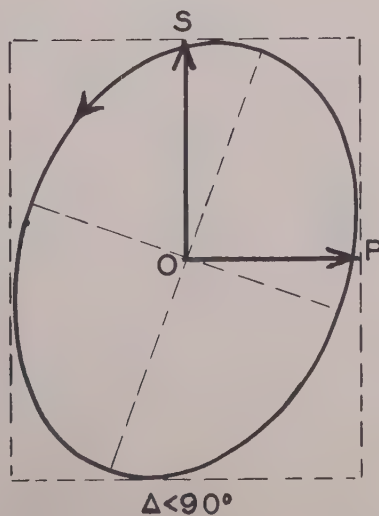
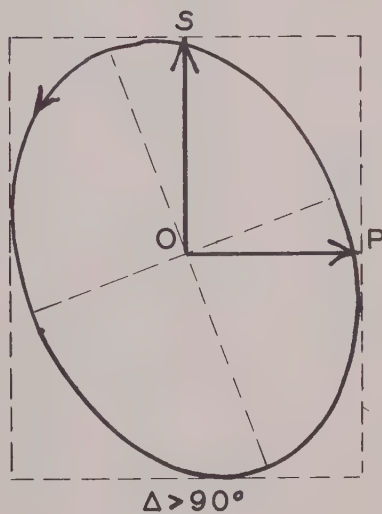
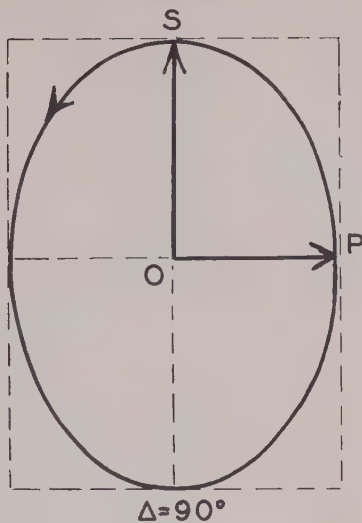
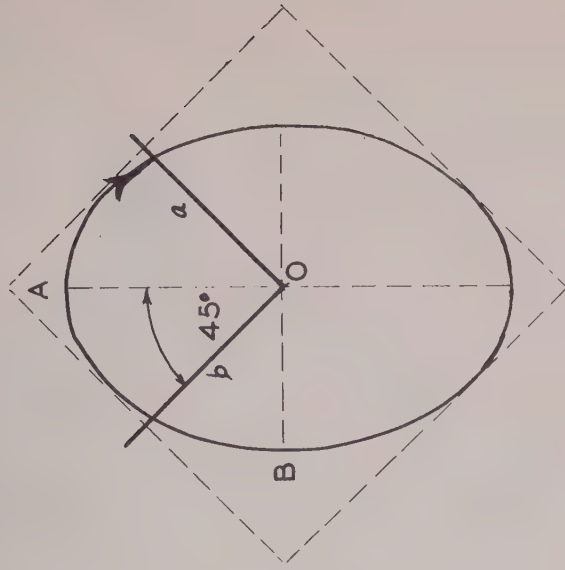
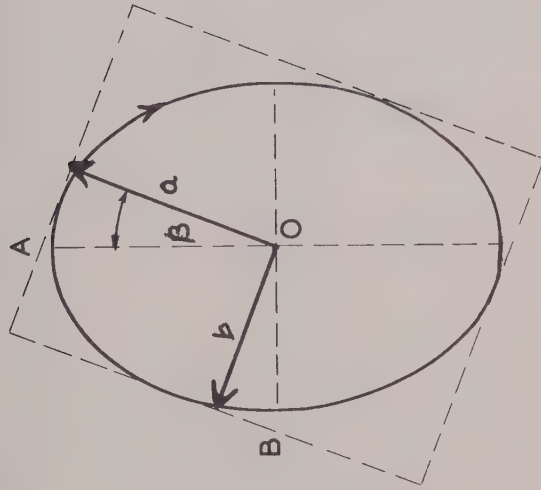


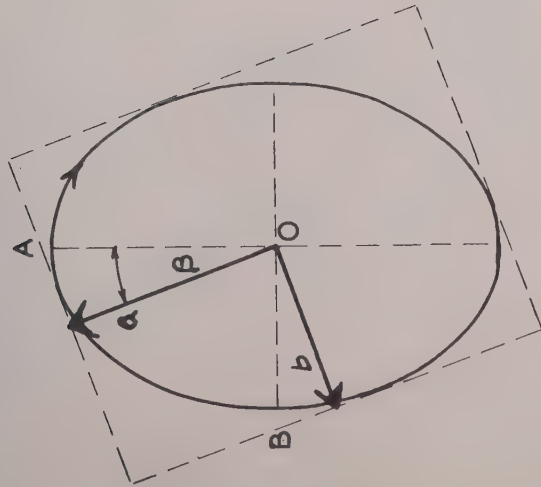
FIGURE 2



$$\begin{aligned} \text{III} \\ \sin \Delta = \sin 2\epsilon \\ \beta = 45^\circ \end{aligned}$$



$$\begin{aligned} \text{II} \\ \Delta < 90^\circ \end{aligned}$$



$$\begin{aligned} \text{I} \\ \Delta > 90^\circ \end{aligned}$$

FIGURE 3

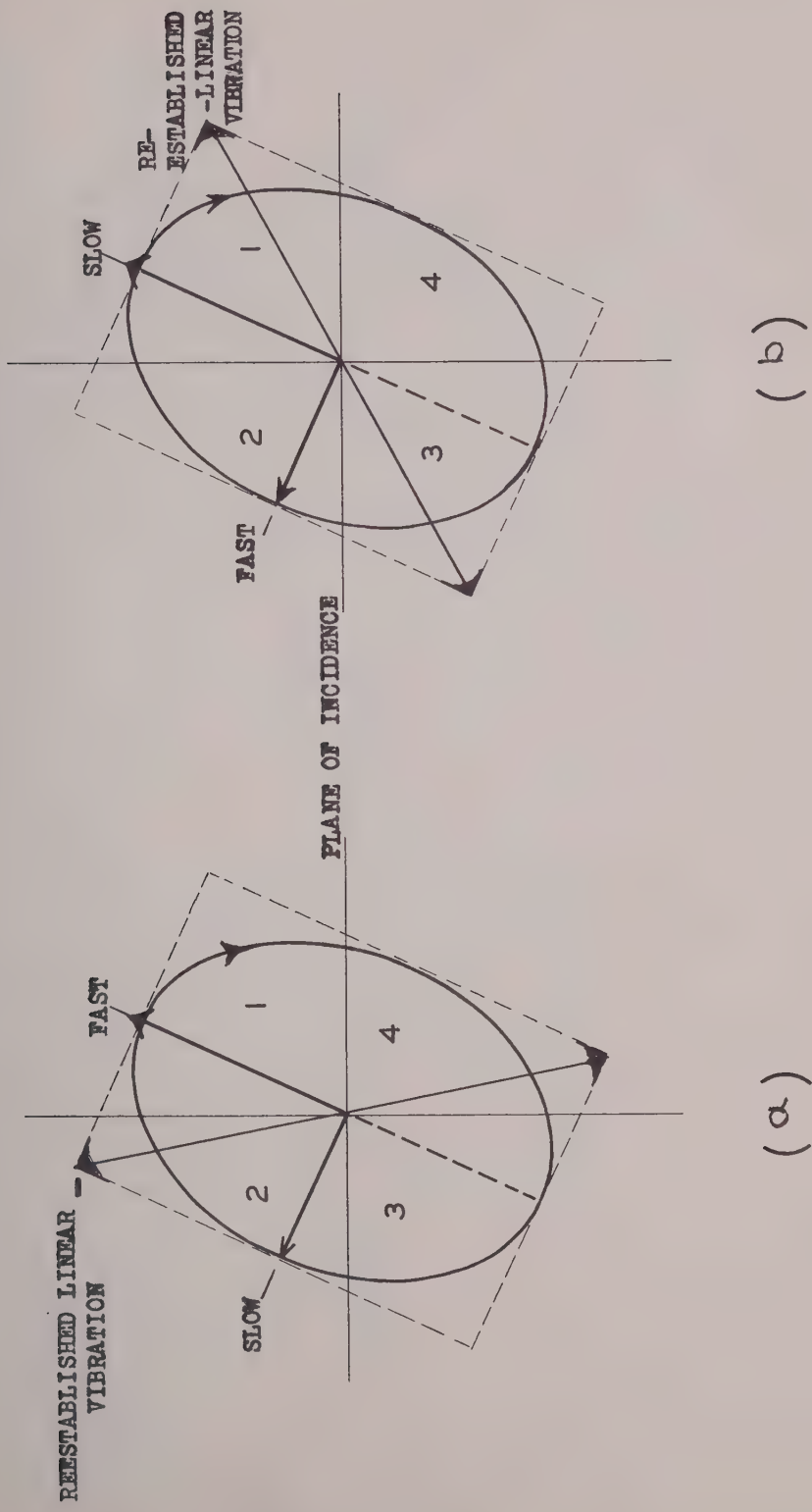


FIGURE 4

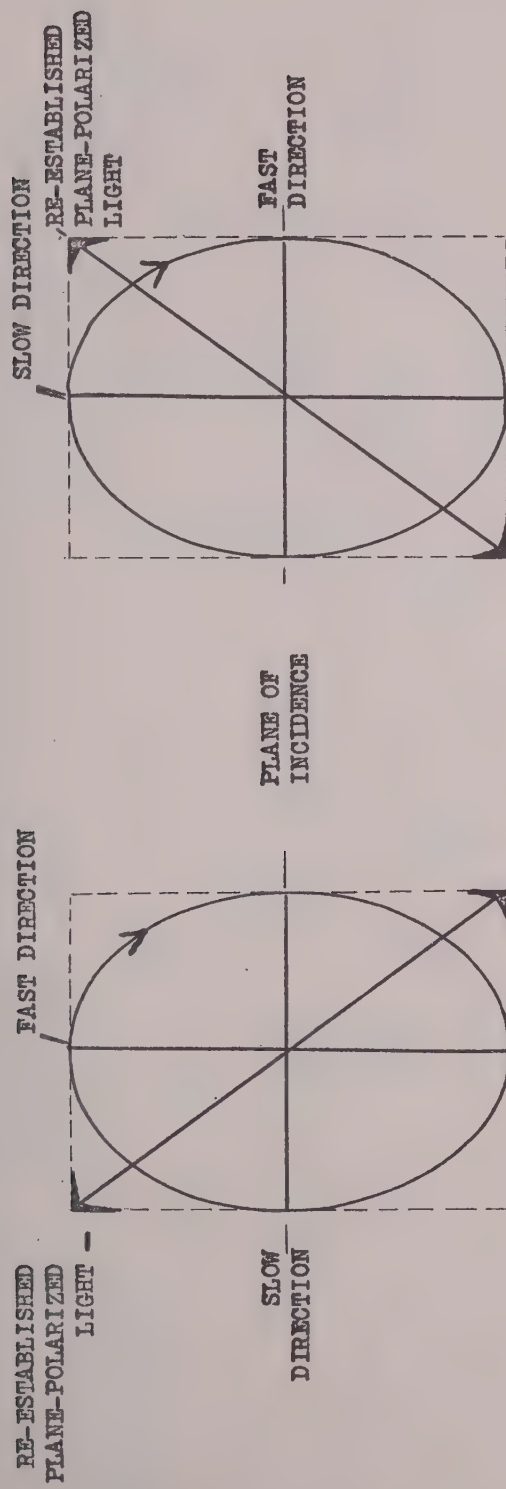


FIGURE 5

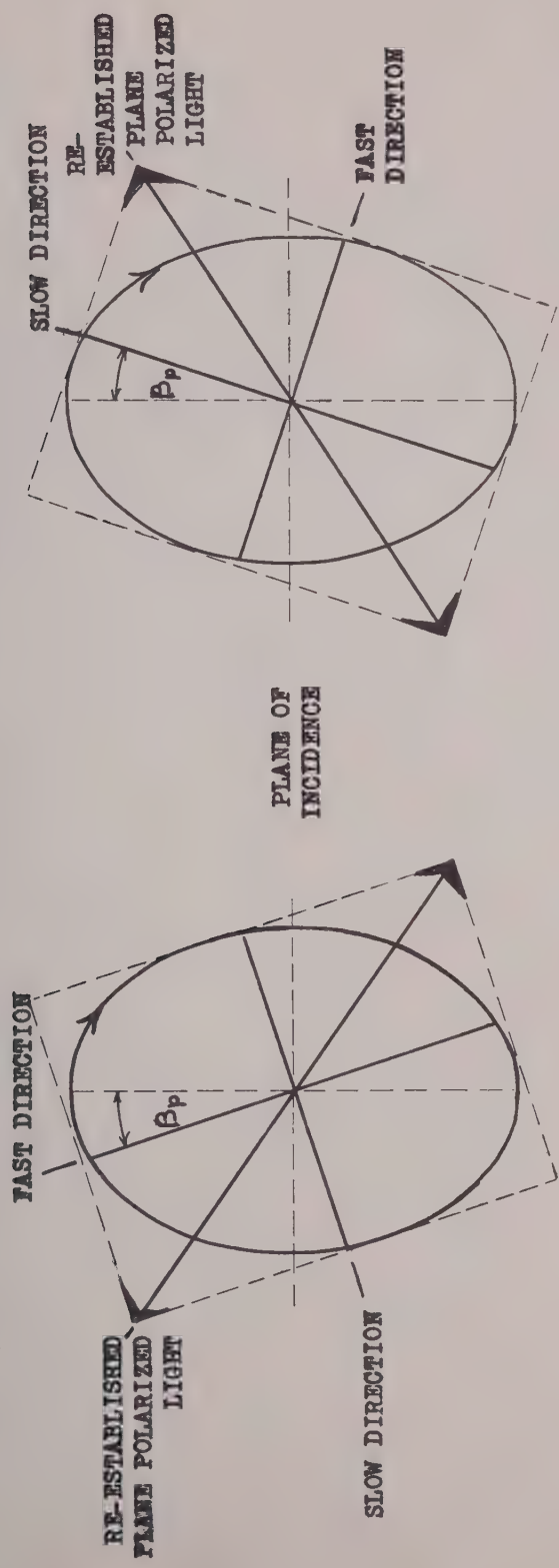
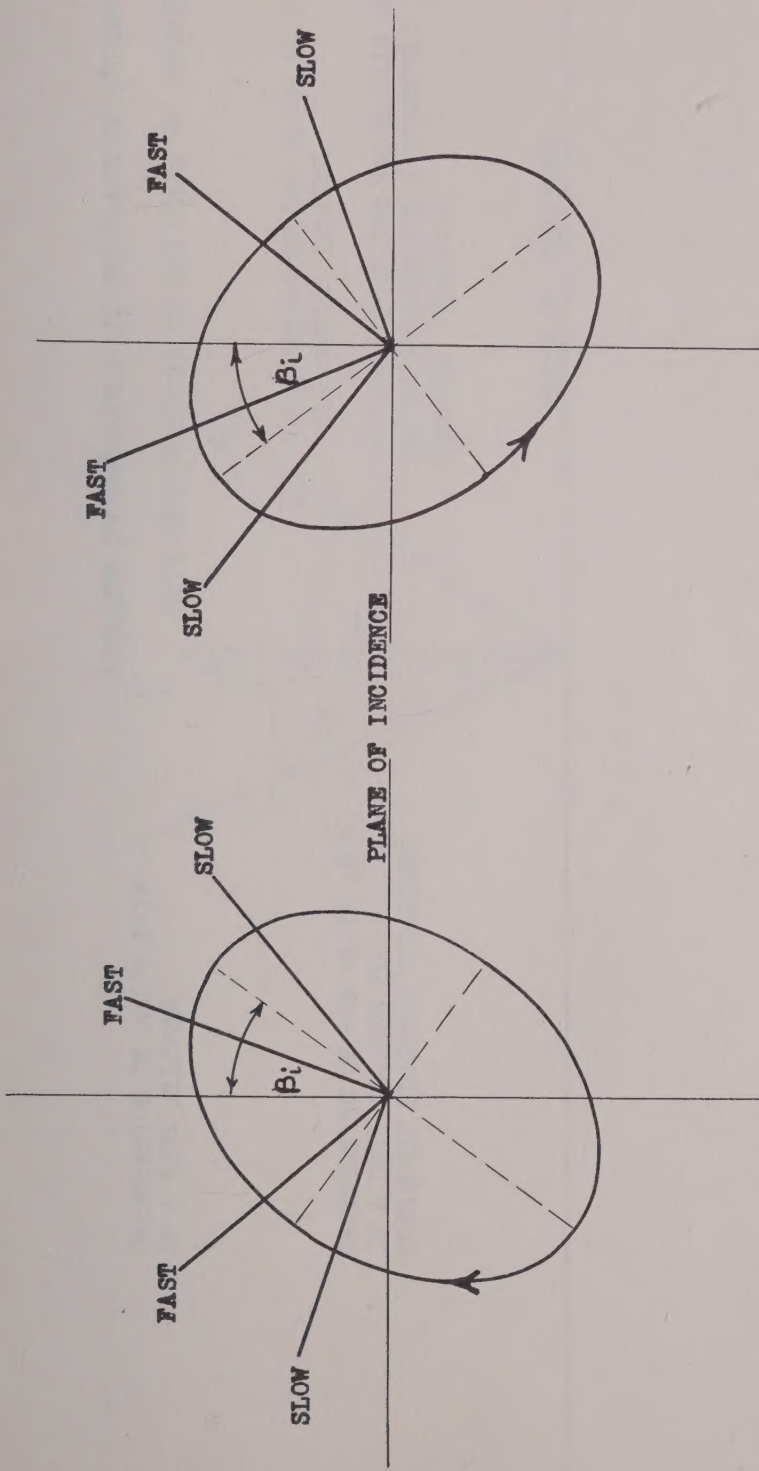


FIGURE 6



INCIDENT VIBRATION
IN THE 1st AND 3rd
QUADRANTS

INCIDENT VIBRATION
IN THE 2nd AND 4th
QUADRANTS

FIGURE 7

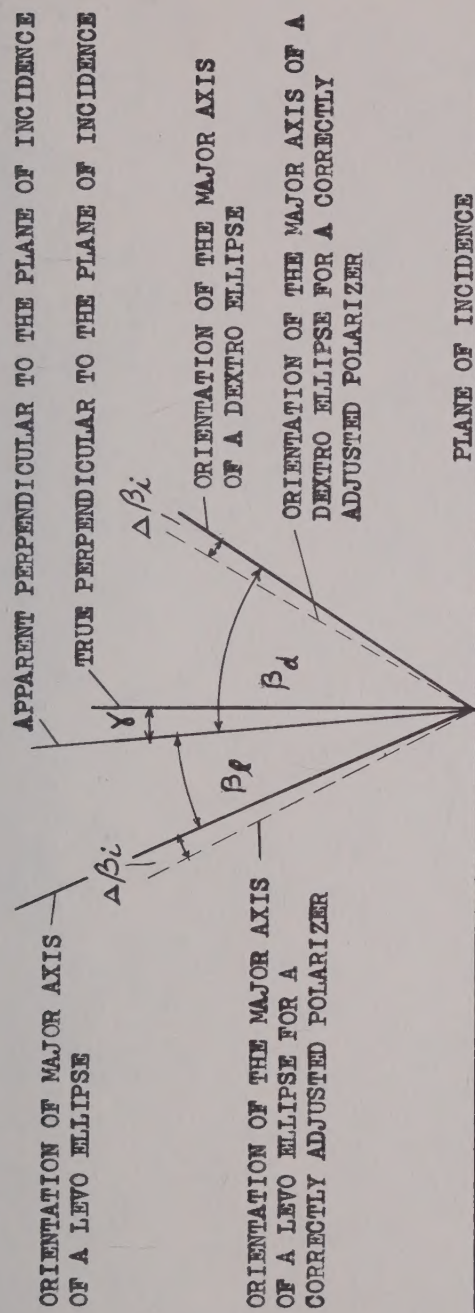


FIGURE 8

2466-1

322.52

311.45

810.00
633.97

176.03

170.
169.81

165.00

34
2 | 334.81
180.00
167.40

12.50

2466-2

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180

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180

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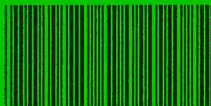
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